Thursday Dec. 14, 2006

SOLUTIONS FINAL MA1160/1161

Fall 2006 — 200 pts

[15] ① $f(-1)=7$ $f'(-1)=-8$ $g(-1)=-6$ $g'(-1)=-2$
$f(1)=3$ $f'(1)=-5$ $g(1)=-3$ $g'(1)=4$
$f(0)=4$ $f'(0)=3$ $g(0)=-1$ $g'(0)=2$

3 pts ⓐ $H'(x)=3f'(x)-2g'(x)$
$H'(0)=3f'(0)-2g'(0)=3(3)-2(2)=9-4=5$

4 pts ⓑ $K'(x)=f'(x)g(x)+f(x)g'(x)$
$K'(0)=f'(0)g(0)+f(0)g'(0)$
$=(3)(-1)+(4)(2)=-3+8=5$

4 pts ⓒ $\frac{dL}{dx}=\frac{df}{dg}\frac{dg}{dx}$ so $L'(x)=f'(g(x))g'(x)$.
$L'(0)=f'(g(0))g'(0)=f'(-1)g'(0)=(-8)(2)$
$=-16$

4 pts ⓓ $M'(x)=\frac{g'(x)f(x)-g(x)f'(x)}{(f(x))^2}$
$M(x)=\frac{g(x)}{f(x)}$: $M'(0)=\frac{g'(0)f(0)-g(0)f'(0)}{(f(0))^2}=\frac{(2)(4)-(-1)(3)}{(4)^2}$
$=\frac{8+3}{16}=\frac{11}{16}=0.6875$

Part I (15 pts) ← page 1

[4] ② $a=0$ $b=10$ $n=5$ $\frac{b-a}{5}=\frac{10-0}{5}=\frac{10}{5}=2$
$\int_0^{10}f(x)\,dx \sim \frac{b-a}{5}[f(0)+f(2)+f(4)+f(6)+f(8)]$
$=2[2+12+30+62+70]=2(44+132)$
$=2(176)=352$

[7] ③

← two trapezoids

4 pts ⓐ $\int_0^4 f(x)\,dx=\frac{3+1}{2}(2)+\frac{1+2}{2}(2)$
$=4+3=7$

Part I

3 pts ⓑ $\frac{1}{4-0}\int_0^4 f(x)\,dx=\frac{1}{4}\int_0^4 f(x)\,dx=\frac{7}{4}=1\frac{3}{4}$
or $=1.75=f_{ave}$ on $[0,4]$ (11 pts) ← page 2

[12] ④ $\int_a^b f(x)dx=5$, $\int_a^b g(x)dx=3$, $\int_a^b f^2(x)dx=16$, $\int_a^b g^2(x)dx=36$
4 pts ⓐ $\int_a^b(4f(x)-3g(x))dx=4(5)-3(3)=20-9=11$
4 pts ⓑ $\int_a^b(4f^2(x)-3g^2(x))dx=4(16)-3(36)=64-108=-44$
4 pts ⓒ $4\left[\int_a^b f(x)dx\right]^2-3\left[\int_a^b g(x)dx\right]^2=4[5]^2-3[3]^2$
$=4(25)-3(9)=100-27=73$

Part I (12 pts) ← page 3

[30] ⑤ ⓐ $f(x)=4x^3+e^{\pi}$ (const), $f'(x)=12x^2+0=12x^2$
6 pts

ⓑ $g(x)=2\cos(8x+3)$ Part II
6 pts $g'(x)=-16\sin(8x+3)$

ⓒ $h(x)=\ln[1+x^3]$
6 pts $h'(x)=\frac{3x^2}{1+x^3}$

ⓓ $P(t)=4e^{t^2}$, $P'(t)=8te^{t^2}$
6 pts

ⓔ $G(x)=\arcsin(e^x)$
6 pts $G'(x)=\frac{e^x}{\sqrt{1-(e^x)^2}}=\frac{e^x}{\sqrt{1-e^{2x}}}$

(30 pts) ← Page 4

[12] ⑥ $g(x)=x^2\sin(x^3)$
ⓐ $g'(x)=2x\sin(x^3)+x^2(3x^2)\cos(x^3)$
6 pts $=2x\sin(x^3)+3x^4\cos(x^3)$

ⓑ $h(x)=\int_0^x\cos[(2t^4)]dt$ Part II
6 pts $h'(x)=\cos[2x^4]$

[6] ⑦ $f(x)=2+3\sin[x+\pi]$
$x=0$ $f(0)=2+3\sin\pi=2+0=2$
$\therefore A=(0,f(0))=(0,2)$ ← pt on
2 pts graph of $y=f(x) \Leftrightarrow x=0$

$f'(x)=0+3\cos(x+\pi)=3\cos(x+\pi)$
2 pts $f'(0)=3\cos\pi=-3=m=$ slope of tangent line
$\therefore y-f(0)=f'(0)(x-0)$
$y-2=-3(x-0)=-3x$
2 pts or
$y=2-3x$ ← eqs for tangent line at $A=(0,2)$

(18 pts) ← page 5

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PART II

[12] (8) (a) $\int (8t^3+3t^2)\,dt$ PART II

6pts

$= \boxed{2t^4 + t^3 + C}$ ← 1 pt

5pts

(b) $\int_1^2 (8t^3+3t^2)\,dt = 2t^4+t^3 \Big|_1^2$

6 pts $= 2(16-1) + (8-1) = 2(15) + 7 = 37$

OR $[2(16)+8] - [2+1] = 40 - 3 = 37$

[17] (9) (a) $g(x) = \frac{12}{x} + 2\cos x$ — we anti-deriv $g(x)$

6pts $\int g(x)\,dx = \int\left(\frac{12}{x} + 2\cos x\right)dx = 12\ln|x| + 2\sin x$

(b) $h(x) = 12\sin(3x) + \frac{1}{\cos^2 x}$

6pts $\int h(x)\,dx = 12\int \sin(3x) + \int \frac{dx}{\cos^2 x}$ $+ \int\frac{dx}{\cos^2 x}$

$= -4\cos(3x) + \tan x$ ← an anti-deriv of $h(x)$

(c) $G(x) = 7^x = e^{x\ln 7}$

5pts $\int G(x)\,dx = \int 7^x\,dx = \frac{1}{\ln 7}(7^x)$

(29 pts) ← PAGE 6

[12] (10) $f(x) = -(x+3)(x-2)^2$

$f'(x) = -9(x+4/3)(x-2)$

$f''(x) = -18(x-1/3)$

(a) $f(x)$: + 0 − 0 − ; x: −3, 2

4pts $f(x)$ is "+" for $x < -3$

(b) $f'(x)$: − 0 + 0 − ; x: −4/3, 2

4pts $f'(x)$ is "+" for $-\frac{4}{3} < x < 2$

(c) $f''(x)$: + 0 − ; x: $\frac{1}{3}$

4pts $f''(x)$ is "+" for $x < \frac{1}{3}$

[10] (11) $s = s(t) = 6t^3 + 8t^2 + 3t + 5$

dist: meters time = secs 3pts

5pts (a) $v = \frac{ds}{dt} = s'(t) = 18t^2 + 16t + 3$ ← 3pts

$t = 1$ sec $v = 18 + 16 + 3 = 37$ m/sec ← 2pts

(b) ave velocity from $t=0$ to $t=2$

5pts given by $\frac{s(2)-s(0)}{2-0}$

$= \frac{6(8) + 8(4) + 3(2) + \cancel{5} - \cancel{5}}{2} = \frac{48+32+6}{2}$

$= 24 + 16 + 3 = 43$ m/sec (22pts) ← PAGE 7

[12] (12) $A = (1,1)$ $y = y(x)$

$x^4 + xy^2 + y^3 = 3$

(a) $4x^3 + 1\cdot y^2 + x\,2y\frac{dy}{dx} + 3y^2\frac{dy}{dx} = 0$

6pts $(2xy + 3y^2)\frac{dy}{dx} = -(4x^3 + y^2)$

$\therefore \frac{dy}{dx} = -\frac{4x^3+y^2}{2xy+3y^2} = y'$

(b) 6pts $m = y'(1) = \frac{dy}{dx}\Big|_{A=(1,1)} = -\frac{4+1}{2+3} = -\frac{5}{5} = -1$

$\therefore y - 1 = -(x-1)$ or $y = 1 - (x-1)$

or $y = 2 - x$ ← eqs of the tangent line at A.

[8] (13) $A = 4\pi r^2$ $r = r(t)$ so $A = A(t)$

$\frac{dA}{dt} = 4\pi(2r)\frac{dr}{dt} = (8\pi r)\frac{dr}{dt}$ ← 4pts

$\frac{dr}{dt} = 3$ cm/min ← 2pts

$r = 5$ cm

$\frac{dA}{dt} = (8\pi)(5)(3)$ cm²/min $= 120\pi$ cm²/min

(20pts) ← PAGE 8

[16] (14) PART II

$f'(x)$: + 0 + 0 − 0 + 0 −
x: −2, 2, 4, 8

$f''(x)$: − 0 + 0 − 0 + 0 −
x: −2, 0, 3, 6

(a) $f(x)$ decreasing $\Leftrightarrow$ $f'(x)$ "−"

4pts $2<x<4$ & $8<x$ (or)
$x \in (2,4) \cup (8,\infty)$

4pts (b) $f(x)$ has local maxs at
$x = 2, 8$

4pts (c) $f(x)$ is concave up $\Leftrightarrow$ $f''(x)$ "+"
$-2<x<0$ and $3<x<6$ (or)
$x \in (-2,0) \cup (3,6)$

4pts (d) $f(x)$ has pts of inflection at
$x = -2, 0, 3, 6$

(16pts) ← PAGE 9

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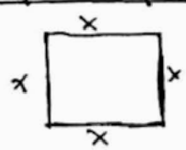
[27] (15) PART II



SQ BASE side X cm

$0<x<\infty$ & $0<y<\infty$ these are physical dimensions
NO TOP ON THE BOX alt y cm

FIXED VOLUME FOR BOX $= V = 4000\text{ cm}^3 = x^2 y$

(a) surface area of box = A = area of
8pts the base + area of the sides
$= x^2 + 4xy = A$

$y = \frac{4000}{x^2} \Rightarrow A = x^2 + \frac{(4\cancel{x})4000}{\cancel{x^2}x}$

$A = x^2 + \frac{16000}{x} = A(x)$

10pts (b) $A'(x) = 2x - \frac{16000}{x^2} = \frac{2x^3 - 16000}{x^2}$
6pts $= \frac{2}{x^2}(x^3 - 8000)$

$A'(x) = 0$ if $x^3 = 8000 = (20)^3$ or
if $x = 20$ ∴ 2pts

4pts
$A'(x)$: (− 0 +
x: 0, 20

2pts

9pts (c) $x = 20\text{ cm}$ ← 3pts

3pts → $A(20) = (20)^2 + \frac{1600\cancel{0}}{2\cancel{0}} = 400 + 800 = 1200\text{ cm}^2$

From (b) signs of $A'(x)$ (1st deriv test)
A has a minimum at $x = 20$ cm
3pts

∴ A has a global minum
of 1200 cm^2 when $x = 20$ cm

(27 pts) ← PAGE 10